

By equating the strain energy in Figs. 1 and 2 the stiffness matrix defined by the relation,  $F = ku$ , is obtained

$$[k] = \beta \alpha^{-1} \beta^T \quad (6)$$

$$[k] = \frac{12}{a^2 + 12g} \frac{EI}{a} \begin{bmatrix} 1 & & & \text{sym} \\ a/2 & a^2/3 + g & & \\ -1 & -a/2 & 1 & \\ a/2 & a^2/6 - g & -a/2 & a^2/3 + g \end{bmatrix} \quad (7)$$

The stiffness matrix given in Eq. (7) is exactly the same as the one obtained in Ref. 1 using a displacement formulation.

The important conclusion from the derivation given in this comment is that by using the flexibility matrix one can include the shear deformation without worrying about the differences between the first derivative of the transverse displacement and the rotation. These differences are distinguished automatically.

### References

- <sup>1</sup>Narayanaswami, R. and Adelman, H.M., "Inclusion of Transverse Shear Deformation in Finite Element Displacement Formulations," *AIAA Journal*, Vol. 12, Nov. 1974, pp. 1613-1614.

## Reply by Authors to M. Baruch

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**M**BARUCH has correctly pointed out that inclusion of transverse shear deformation in finite element force (flexibility) formulations does not require choosing between the first derivatives of the transverse displacements and the rotations of the normals to the neutral surface of the finite element as degrees of freedom. Once the flexibility matrix has been derived in terms of an appropriate set of forces and moments and inverted to form the stiffness matrix, the generalized coordinates associated with the rows and columns of the stiffness matrix are guaranteed to be the correct variables. Incidentally, the flexibility matrix derived by Baruch may also be found in Ref. 1.

The point of Ref. 2 as stated in the conclusions was that, in contrast to previous assertions,<sup>3,4</sup> the finite element *displacement* approach proceeds by a straight-forward energy minimization to yield the correct element stiffness matrix even when transverse shear deformation is included. To obtain the correct result by the displacement approach, however, one must use the correct rotational degrees of freedom, namely the rotations of the normals to the middle surface.

### References

- <sup>1</sup>Przemieniecki, J. S., *Theory of Matrix Structural Analysis*, McGraw-Hill, New York, 1968, p. 175.  
<sup>2</sup>Narayanaswami, R. and Adelman, H. M., "Inclusion of Transverse Shear Deformation in Finite Element Displacement Formulations," *AIAA Journal*, Vol. 12, Nov. 1974, pp. 1613-1614.

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- <sup>3</sup>Severn, R. T., "Inclusion of Shear Deflection in the Stiffness Matrix for a Beam Element," *Journal of Strain Analysis*, Vol. 5, April 1970, pp. 239-241.

- <sup>4</sup>Irons, B. M. and Razzaque, A., "Introduction of Shear Deformation into a Thin Plate Displacement Formulation," *AIAA Journal*, Vol. 11, Oct. 1973, pp. 1438-1439.

## Comment on "Stability of a Spinning Satellite with Flexible Antennas"

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**A** RECENT paper by Dong and Schlack<sup>1</sup> presents a stability analysis of a spinning flexible satellite via the assumed modes method, whereby the displacements of the flexible parts are represented by series of space-dependent admissible functions multiplied by time-dependent generalized coordinates. As admissible functions they use the fixed-base cantilever modes. Neither the procedure nor the results are new, however.

The problem considered in Ref. 1 was first investigated by Meirovitch and Nelson,<sup>2</sup> who used the assumed modes method in conjunction with an infinitesimal analysis to test the stability of precisely the same mathematical model as that of Ref. 1. In fact, the parameter plot of Ref. 1 (Fig. 2) was originally presented in Ref. 2 (Fig. 2). The stability analysis of both a torque-free spinning flexible satellite and a gravity-gradient stabilized flexible satellite was performed by Meirovitch and Calico<sup>3,4</sup> via the Liapunov direct method in conjunction with the assumed modes method (in addition to the method of integral coordinates and the method of testing density functions). Although Ref. 3 is concerned with a more complicated mathematical model than that of Ref. 2 (and hence than that of Ref. 1), in the sense that the system contains four radial booms in addition to the two axial booms, it does consider also the mathematical model of Ref. 2 for comparison purposes.

The authors of Ref. 1 imply that the ability to derive closed-form stability criteria in terms of infinite series represents a new development. A careful examination of both Refs. 3 and 4, however, reveals that the closed-form stability criteria derived in these references by the assumed modes method are indeed in terms of infinite series. In fact, they correspond exactly to those of Ref. 1, as for the assumed modes method the bounding properties of Rayleigh's quotient need not be used and were in fact not used. The Hessian matrix tested for sign definiteness can be shown<sup>5</sup> to have the form

$$[H]_E = [I + b_i \delta_{ij}]$$

where  $b_i (i=0,1,2,\dots)$  are real numbers depending on the system parameters. Using Sylvester's criterion, and con-

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